Journal of Sound and Vibration (1997) **199**(2), 207–221

VIBRATION ANALYSIS OF A NON-LINEAR COUPLED TEXTILE–ROTOR SYSTEM WITH SYNCHRONOUS WHIRLING

R.-F. Fung and J.-S. Shieh

Department of Mechanical Engineering, Chung Yuan Christian University, Chung-Li, Taiwan 32023, *Republic of China*

(*Received* 28 *April* 1995, *and in final form* 19 *March* 1996)

In this paper a dynamic formulation is presented for the coupled textile–rotor system. Both the partial differential equation for the textile thread and the ordinary differential equation for the rotor whirling vibration are derived by Hamilton's principle. When the textile is wound either on or off the rotor, the mass, inertia and unbalance magnitude of the rotor change with time, and also the length of the textile is time-dependent. The Galerkin method is used with a time-dependent basis function to determine approximate solutions. Finally, numerical examples are presented to show the effects of the angular rotational speed, the shaft stiffness and the non-linear terms on the transient amplitudes of the coupled system.

1. INTRODUCTION

Usually, mechanical designers consider the textile and rotor independently in two distinct design processes. For example, much work has been done on the dynamic response of a textile under specified conditions, while a number of studies have been devoted to the steady state response and stability of the rotor. However, the dynamic behavior of these two systems is interrelated when the textile is wound on or off by the rotor.

Much research on the vibration behavior of string-like problems has been performed previously [1–3]. These studies consider a string system of fixed length with no axial motion. Some work [4, 5] has recently appeared in the literature concerning the vibration and dynamic stability of an axially moving beam. Although the string-like systems exhibit movement, the interest of such studies is still in the fixed length condition. Wang [6] studied the global dynamic behavior of a non-linear model of axially moving bands with finite end curvatures under high speed operating conditions. Wang [7] also studied an integrated chain drive system which coupled the sprocket motion with the transverse and longitudinal vibrations of the axially moving chain spans.

For the problem of string vibration with time-varying length, Kotera and Kawai [8] analyzed the free vibrations by Laplace transformation. Fung and Cheng [9] studied the free vibration of a string–slider system with non-linear coupling. It should be noted that the concepts of natural modes and frequencies become meaningless, because as the length of the string varies the natural frequencies become time-dependent, and the independence of the natural modes of oscillation is lost. While in theory the motion of a string of variable length can be described to any desired degree of approximation by an infinite system of differential equations, the mathematical difficulty usually becomes prohibitive for all but

0022–460X/97/020207 + 15 $25.00/0/sv960623

the first few orders of approximation. As far as string vibration is concerned, little work has appeared on coupled oscillation, both from the point of view of the theoretical formulation of the problem and the analysis of the structural behavior.

A related problem involving the oscillations and the influence of the reactive force on the motion of a textile machine rotor on which the textile is wound up was presented in a series of papers by Cventicanin [10–13]. The dynamics of a rotor with variable mass are given by Bessonov [14]. Usually, the rotor consists of a disk which is symmetrically mounted at the middle of the shaft, and the elastic force in the shaft is assumed to be non-linear. The mass of the shaft is negligible in comparison to that of the disk. The mass of the rotor is varying due to winding on of the band. Cventicanin [15] studied the textile machine rotor, in which the angular velocity was constant. The function of the rotor was to wind on a band of textile material. The free vibration in the non-resonant case was analyzed and the solution was found by use of the analytical method of multiple scaling. With a new procedure based on the Krylov–Bogoliubov method, Cventicanin [16] observed the dynamic behavior of a rotor with variable parameters and small non-linearity.

The purpose of this paper is to investigate the qualitative features of the non-linear equations of the whole system, which consists of an axially moving textile and a whirling rotor. The coupled dynamic equations for the textile–rotor system are derived from Hamilton's principle. We adopt the concept of Cventicanin [10] and consider the Jeffcott rotor with variable mass, inertia and unbalance magnitude. The organization of the paper is as follows. In section 2, the coupled model for the textile–rotor system, including both the motion of the textile and vibration of the rotor, is described and formulated. In section 3, we define the non-dimensional variables and use Galerkin's method, with a time-dependent basis function, to determine the approximate solution. In section 4, the numerical results are presented to show the effect of the non-dimensional parameters on the physical characteristics of the coupled system.

2. EQUATION OF MOTION

2.1. GEOMETRY

The coupled textile–rotor system is shown in Figure 1. Two fixed co-ordinate systems, xoy and XOY are used to describe the dynamic configuration. The Jeffcott rotor model investigated here is similar to that used by Vance and Lee [16]. It consists of a single, centrally located, unbalanced disk and an orthotropically elastic shaft, running in two rigid bearings. The shaft is assumed to be massless as compared with the large massive disk. The bearing supports are assumed to be rigid, with the shaft providing all the flexibility. The generalized co-ordinate r is chosen to describe the whirling oscillation of the rotor system which has constant angular velocity ω. Since the rotor is whirling, the textile length is time-dependent. The transverse vibration of the textile is in the interval $0 \leqslant x \leqslant l(t)$. The textile is subject to an initial tension T and has simple supports as the boundary conditions at $x = 0$ and $l(t)$.

Since the connection point $x = l(t)$ is common to the disk and textile, this point on the textile has the same velocity and acceleration as that on the disk in the tangential direction. When the textile is wound on, the disk rotates in a clockwise direction and the shaft angular velocity ω is negative.

2.2. VIBRATION OF TEXTILE

The function of the rotor is to wind the textile on or off, so the rotor mass is variable. In Figure 1, $R(t)$ is the radius of the disk, and ϕ is the rotary angle. The textile is subject

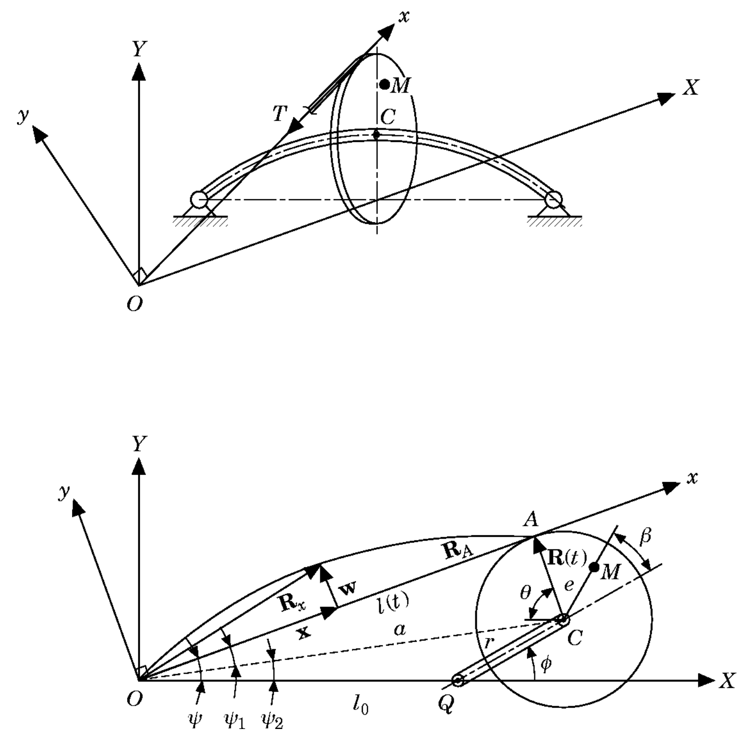

Figure 1. The physical configuration.

to an initial tension T, so this force acts along the tangent line of the disk in the undeformed configuration. The point $x = l(t)$ is one connection point between the textile and the rotor, and $\mathbf{R}_A(t)$ and $\mathbf{R}(t)$ are perpendicular to each other at point A. Since the disk has a whirling motion, the length is time-dependent. From the geometry of Figure 1, the time-varying length of the textile is given by

$$l(t) = \sqrt{l_0^2 + r^2 + 2l_0 r \cos\phi - R^2(t)}. \tag{1}$$

2.3. WHIRLING ROTOR

The rotor is modelled as a rigid disk mounted on a massless shaft, which is supported by two perfect rotating bearings. During winding of the textile on or off the disk, the effective mass and radius of the disk vary. The mass $m(t)$ and radius $R(t)$ are assumed to be as given in the paper by Bessonov [14] and Cventianin [10]:

$$m(t) = m_0 - R_1\rho\omega t, \qquad R(t) = \left(R_0^2 - \frac{R_1 h\omega t}{\pi}\right)^{1/2}, \tag{2, 3}$$

where m_0 and R_0 are, respectively, the mass and radius of the disk without textile, ω is the angular velocity of the rotor and has a negative value for the textile to be wound up, t is time, $R_1 = R_0 + h/2$, and h is the average thickness of the textile. The magnitude of the imbalance is given by the distance $e = CM$, where C is the geometric center of the disk and M is its mass center, as given by Bessonov [14]:

$$e(t) = -\frac{2}{m_0} R_1^2 \rho \sin\frac{\omega t}{2}. \tag{4}$$

The co-ordinate r gives the magnitude of shaft deflection, and the time derivative $\dot{\phi}$ gives the whirling speed. The instantaneous angular location of the imbalance with respect to the plane of shaft bending is given by β, which remains constant when the rotor is in synchronous vibration. The amplitudes of synchronous vibration usually indicate a rotor imbalance problem.

For the case that the whirl speed equals the shaft speed, $\dot{\phi} = \omega$, and the governing equation can be written as (the derivations are detailed in Appendix A)

$$w_{tt} + 2\dot{x}w_{xt} + \ddot{x}w_x - \dot{\psi}^2 w - \left(\frac{T}{\rho} - \dot{x}^2\right)\omega_{xx} - \tfrac{3}{2}\frac{EA}{\rho}\,w_x^2 w_{xx} = -3\dot{x}\ddot{\psi}, \qquad 0 < x < l(t), \quad (5a)$$

$$\begin{aligned}\ddot{r} + \left[\frac{k_x}{m(t)}\cos^2\beta + \frac{k}{m(t)}\sin^2\beta - \omega^2\right]r = {} & e\omega^2\cos\beta + \dot{e}\omega\sin\beta - g\sin\beta \\ & - \frac{T}{m(t)}[\cos\psi\cos\phi - \sin\psi\sin\phi] \\ & - \frac{(r + l_0\cos\phi)}{2m(t)l(t)}[\rho\dot{x}^2 + \rho\dot{x}^2 w_x^2(l(t), t] \\ & - Tw_x^2(l(t), t) - \tfrac{1}{4}EAw_x^4(l(t), t), \end{aligned} \quad (5b)$$

and the boundary conditions are

$$w(0, t) = 0, \qquad w(l(t), t) = 0, \quad (6a, b)$$

where

$$\dot{x} = -R(t)\dot{\phi}, \qquad 0 < x < l(t) \quad (7a)$$

$$\ddot{x} = -\dot{R}(t)\dot{\phi} - R(t)\ddot{\phi}, \qquad 0 < x < l(t). \quad (7b)$$

From the governing equations (5a) and (5b) and the velocity and acceleration (7a) and (7b) of the textile, the following observations are made.

(i) The mass, inertia and imbalance magnitude of the rotor are time-varying when the textile is wound on or off. Therefore the non-linear governing equations (5a) and (5b) include the time-dependent mass $m(t)$, the inertia $I(t)$ and the eccentricity $e(t)$.

(ii) In this paper, the longitudinal elastic deformation of the textile is neglected, so every point along the textile has the same axial travelling velocity $\dot{x}$ and acceleration $\dot{x}$, which are given by equations (7a) and (7b).

(iii) The axial travelling velocity $\dot{x}$ is a positive value and the textile moves along the positive x-axis direction. In the case of constant angular velocity, the radius $R(t)$ of the disk and the axial velocity $\dot{x}$ of the textile are non-linear functions of time. Then $\dot{R}(t)$ is not equal to zero, and the axial travelling acceleration $\dot{x}$ also exists.

(iv) The non-homogeneous terms, $-3\dot{x}\dot{\psi} - x\ddot{\psi}$, in equation (5a) are the whirling effects of the rotor on textile vibration. The terms including $w_x(l(t), t)$ in equation (5b) are the end effects at $x = l(t)$ of the textile vibration on the rotor whirling.

(v) The terms containing EA in equations (5a) and (5b) are due to the geometric non-linearity of the textile. If they are neglected for small amplitude transverse vibration of the textile, the governing equation (5a) becomes linear. However, equation (5b) is still non-linear, due to the boundary at $x = l(t)$.

(vi) The emphasis is placed on the moving boundary condition of the coupled textile–rotor system. The connection point $x = l(t)$ is not specified and its position moves with time. The boundary position $x = l(t)$ will be solved simultaneously with equations (5a) and (5b).

3. METHOD OF SOLUTION

3.1. DIMENSIONLESS FORM OF GOVERNING EQUATIONS

For convenience in determining the influence of the coupled system parameters, we define the following non-dimensional variables:

$$W = w/l_0, \qquad \xi = x/l_0, \qquad \tau = c_2 t/l_0, \qquad \bar{l} = l(t)/l_0, \qquad \bar{g} = g l_0/c_2^2, \qquad \eta = c/c_2,$$

$$\beta_1 = c_1/c_2, \qquad M = \rho l_0/m(t), \qquad \bar{r} = r/l_0, \qquad \bar{e} = e/l_0, \qquad \bar{R} = R(t)/l_0,$$

$$\bar{I} = I(t)/m(t)l_0^2, \qquad \Omega = l_0/c_2\omega, \qquad \Omega_x^2 = k_x l_0/m(t)c_2^2, \qquad \Omega_y^2 = k_y l_0/m(t)c_2^2,$$

where

$$c_1 = \sqrt{EA/\rho}, \qquad c_2 = \sqrt{T/\rho}.$$

The latter are the wave velocities of the textile in the longitudinal and transverse directions respectively. Then the equations of motion in non-dimensional form are

$$W_{\tau\tau} + 2\eta W_{\xi\tau} + \xi_{\tau\tau} W_\xi - \psi_\tau^2 W + (\eta^2 - 1) W_{\xi\xi} - \tfrac{3}{2}\beta_1^2 W_\xi^2 W_{\xi\xi} = -3\eta\psi_\tau - \xi\psi_{\tau\tau}, \qquad 0 < \xi < \bar{l}, \tag{8a}$$

$$\begin{aligned} &\bar{r}_{\tau\tau} + [\Omega_x^2 \cos^2\beta + \Omega_y^2 \sin^2\beta + \Omega^2]\bar{r} \\ &= \bar{e}\Omega^2 \cos\beta - \bar{g}\sin\phi - M W_\xi(\bar{l},\tau)\psi_\tau - \frac{M}{2}(\bar{r} + \cos\phi)\psi_\tau^2 \\ &-\frac{\bar{r} + \cos\phi}{2\bar{l}}\left[M\xi_\tau^2 + M\xi_\tau^2 W_\xi^2(\bar{l},\tau) - \frac{M}{2} W_\xi^2(\bar{l},\tau)\right. \\ &\left.+\frac{M}{4}\beta_1^2 W_\xi^4(\bar{l},\tau)\right] - M(\cos\psi\cos\phi - \sin\psi\sin\phi - \bar{C}_r\bar{R}) \end{aligned} \tag{8b}$$

and the non-dimensional boundary conditions are

$$W(0,\tau) = 0, \qquad W(\bar{l},\tau) = 0. \tag{9a, b}$$

3.2. APPROXIMATE SOLUTION BY THE GALERKIN METHOD

Initially, the spatial dependence must be eliminated from the equations of the coupled system to yield a set of ordinary differential equations in time, which can be solved for the system response. In view of the non-linear nature of the equations in the previous section, Galerkin's method is used here to separate the spatial co-ordinates from the temporal variable.

The approximate solution derived in this section is based on a Galerkin approximation with a time-dependent basis function. This was also used by Wang and Wei [17] to analyze vibrations in a moving flexible robot arm, by Yuh and Young [18] to study the dynamic modeling of an axially moving beam in rotation, and by Fung and Cheng to investigate the free vibration of a non-linear coupled string–slider system with a moving boundary.

Based on this method, the forms of the displacements are assumed to satisfy the geometric boundary conditions of the textile; that is,

$$W(\xi, \tau) = \sum_{n=1}^{\infty} \varphi_n(\xi, \tau) q_n(\tau), \qquad 0 < \xi < \bar{l}, \tag{10}$$

where $q_n(\tau)$ are the generalized co-ordinates and the co-ordinate functions of the space variable are

$$\varphi_n(\xi, \tau) = \alpha_n(\tau) \sin[\Omega_n(\tau)\xi], \qquad n = 1, 2, 3, \ldots, \tag{11}$$

in which

$$\Omega_n(\tau) = n\pi/\bar{l}, \quad \alpha_n(\tau) = \sqrt{2/\bar{l}}, \qquad n = 1, 2, 3, \ldots, \tag{12, 13}$$

Since the spatial domain is time-dependent, both the eigenfunction $\varphi_n(\xi, \tau)$ and its corresponding eigenvalue $\Omega_n(\tau)$ are time-dependent.

Substituting equations (10) and (11) into (8a) and (8b), taking inner products and making use of the orthogonality property, we have a system comprised of an infinite number of non-linear time-varying ordinary differential equations:

$$\ddot{q}_m + \sum_{n=1}^{\infty} [a_{mn}(\bar{l}, \dot{\bar{l}}\dot{q}_m + b_{mn}(\bar{l}, \dot{\bar{l}}, \ddot{\bar{l}})q_m] - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} c_{mijk}(\bar{l}) q_i q_j q_k = -G_m - H_m, \tag{14a}$$

$$\ddot{r} + a^r(\bar{l})\dot{r} = \sum_{n=1}^{\infty} b_n^r(\bar{l}) q_n + \sum_{n=1}^{\infty}\sum_{i=1}^{\infty} [c_{ni}^r(\bar{l}) + d_{ni}^r(\bar{l})] q_n q_i + \sum_{n=1}^{\infty}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} e_{nijk}^r(\bar{l}) q_n q_i q_j q_k + f^r(\bar{l}), \tag{14b}$$

where $m, n, i, j, k = 1, 2, \ldots$ are the modes considered in the string system, $\dot{q} = \mathrm{d}q/\mathrm{d}\tau$, $\dot{r} = (\mathrm{d}r/\mathrm{d}\tau)$ and

$$a_{mn}(\bar{l}, \dot{\bar{l}}) = 2A_{mn}(\bar{l}, \dot{\bar{l}}) + 2\eta B_{mn}(\bar{l}),$$

$$b_{mn}(\bar{l}, \dot{\bar{l}}, \ddot{\bar{l}}) = -\psi_\tau^2 \delta_{mn} + C_{mn}(\bar{l}) + D_{mn}(\bar{l}, \dot{\bar{l}}, \ddot{\bar{l}}) + 2\eta E_{mn}(\bar{l}, \dot{\bar{l}}) + (\eta^2 - 1) F_{mn}(\bar{l}),$$

$$c_{mijk}(\bar{l}) = \tfrac{3}{2}\beta_1^2 N_{mijk}(\bar{l}), \qquad G_m = 3\eta\psi_\tau \frac{\sqrt{2\bar{l}}}{m\pi}[1 - (-1)^m], \qquad H_m = \psi_{\tau\tau}\frac{\sqrt{2\bar{l}^3}}{m\pi}(-1)^m.$$

The details of $A_{mn}(\bar{l}, \dot{\bar{l}})$, $B_{mn}(\bar{l})$, $D_{mn}(\bar{l}, \dot{\bar{l}}, \ddot{\bar{l}})$, $E_{mn}(\bar{l}, \dot{\bar{l}})$, $F_{mn}(\bar{l})$, $N_{mijk}(\bar{l}, \dot{\bar{l}}, \ddot{\bar{l}})$, $a^r(\bar{l})$, $b_n^r(\bar{l})$, $c_m^r(\bar{l})$, $d_m^r(\bar{l})$, $e_{nijk}^r(\bar{l})$ and $f^r(\bar{l})$ are given in Appendix B.

4. NUMERICAL RESULTS AND DISCUSSION

Since the amplitudes of the textile–rotor system are governed by two non-linearly coupled ordinary differential equations, (14a) and (14b), an exact solution is not possible. The examples given here are chosen to study the coupling effect on the transient vibrations of the textile–rotor system. The parameter values are $T = 100$ N, $\rho = 1$ kg/m, $l_0 = 1$ m, $m_0 = 4{\cdot}95$ kg, $R_0 = 0{\cdot}1$ m and $h = 0{\cdot}02$ m, and the values of the rotor stiffness are chosen as $k_x = k_y = k$, so that we have $\Omega_x = \Omega_y$.

In the initial configuration, $t = 0$, $m(0) = m_0$, $R(0) = R_0$ and $\Omega_r^{(0)} = \sqrt{(k/m_0) - \omega^2} l_0/c_2$ is the natural frequency of the rotor. ω is its angular velocity and has a negative value when the textile is being wound on, and $\Omega_1^{(0)} = \pi$ is the first mode frequency of the textile. We

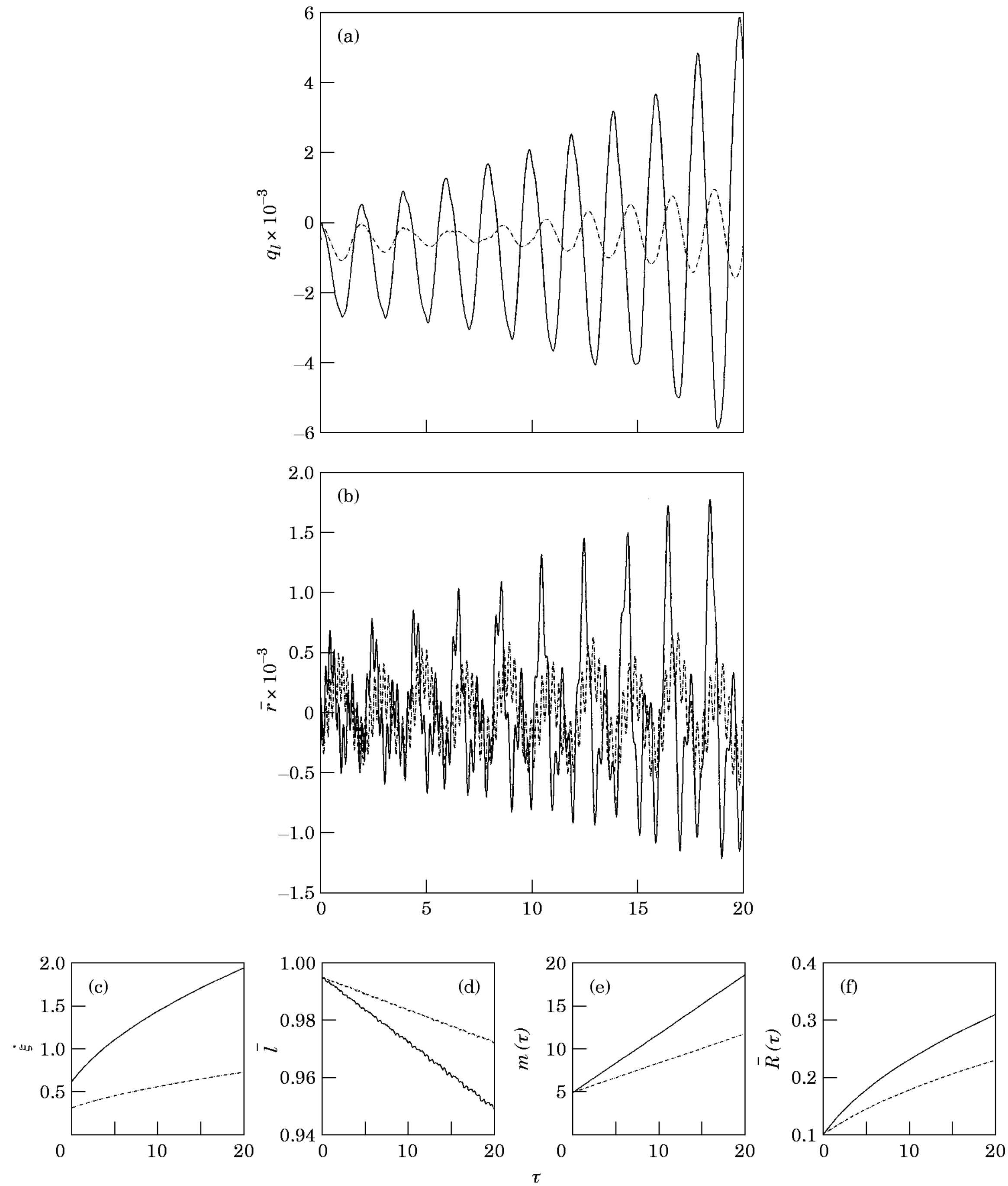

Figure 2. The influence of the angular speed of the rotor on the transient amplitudes for the case $\Omega_r^{(0)}/\Omega_1^{(0)} = 10$: ·—·—·—, $\Omega = -\pi$; ——, $\Omega = -2\pi$. (a) The non-dimensional amplitude of the textile. (b) The non-dimensional amplitude of the rotor. (c) The time-dependent axial velocity $\dot{\xi}$ of the textile. (d) The time-dependent length $\bar{l}$ of the textile. (e) The time-dependent mass $m(\tau)$ of the disk. (f) The time-dependent radius $\bar{R}(\tau)$ of the disk.

consider the textile and rotor with the following initial conditions of the dimensionless variables:

$$q_i(0) = 0, \quad \dot{q}_i(0) = 0, \qquad i = 1, 2, 3,$$
$$\bar{r}(0) = \dot{\bar{r}}(0) = 0.$$

The Runge–Kutta numerical method has been used with these initial conditions and a specified accuracy of 10^{-9} to integrate equations (8a) and (8b). The resulting transient curves are shown in Figures 2–5. The influence of angular speed of the rotor on the transient amplitudes is shown in Figure 2. In Figures 2(a) and (b) are shown the non-dimensional amplitudes as functions of non-dimensional time. These curves were obtained by setting the non-dimensional rotary angular velocity Ω of the shaft to $-\pi$ and

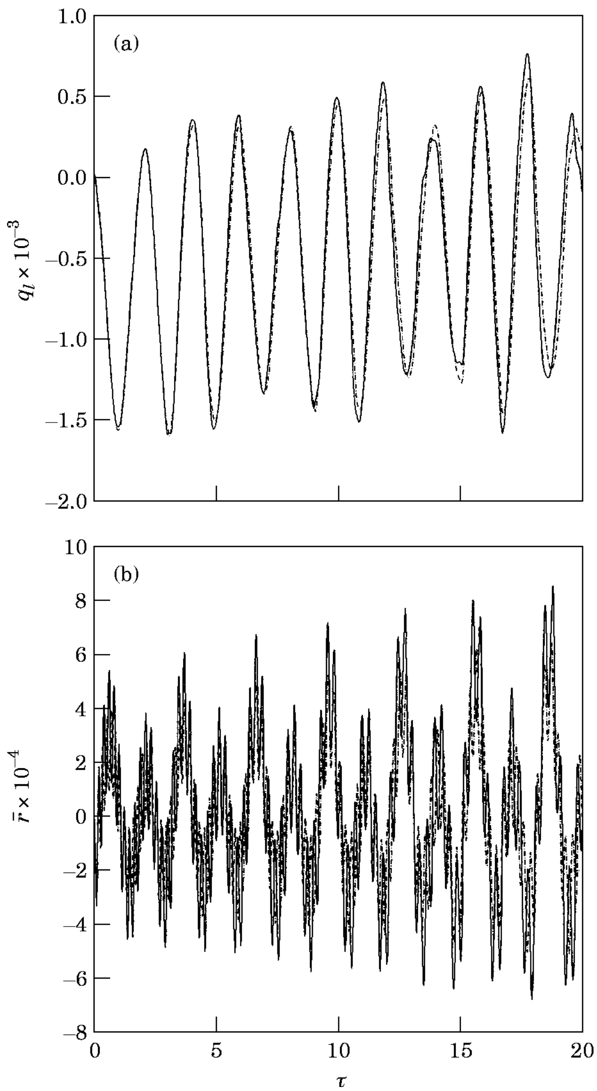

Figure 3. The non-linear effects of the transient amplitudes for the case $\Omega_r^{(0)}/\Omega_1^{(0)} = 10$, $\Omega = -\frac{4}{3}\pi$. (a) The non-dimensional amplitude of the textile: ·–·–·– linear, $\beta_1 = 0$; ——, non-linear, $\beta_1 = 30$. (b) The non-dimensional amplitude of the rotor: ·–·–·–, linear, $\beta_1 = 0$; ——, non-linear, $\beta_1 = 30$.

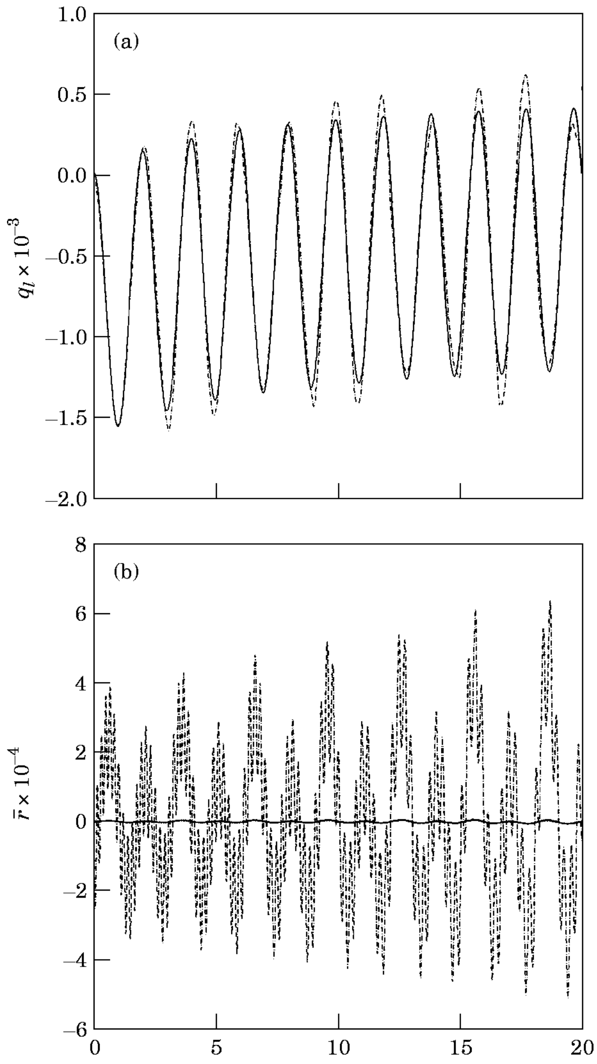

Figure 4. The influence of the shaft stiffness on the transient amplitudes for the case $\Omega = -\frac{4}{3}\pi$: ·–·–·–, $\Omega_r^{(0)}/\Omega_1^{(0)} = 10$; ——, $\Omega_r^{(0)}/\Omega_1^{(0)} = 100$. (a) The non-dimensional amplitude of the textile. (b) The non-dimensional amplitude of the rotor.

-2π respectively, and hence to the first and second mode frequencies of the textile in the initial configuration. It is found that the amplitudes at $\Omega = -2\pi$ (parametric resonance) are larger than those at $\Omega = -\pi$ (harmonic resonance), and both increase with time.

In Figure 2(c) it is shown that the axial travelling velocity $\dot{\xi}$ is not constant, and therefore a travelling acceleration exists in the textile being wound on. The time-dependent length of the textile is given by equation (1), which includes the whirling amplitude $r(t)$ of the rotor. Thus, the time-dependent length of textile has to be determined using equations (14a) and (14b) simultaneously. In Figure 2(d) it is shown that the textile length $\bar{l}$ decreases with time. In Figures 2(e) and (f) it is shown that the mass $m(\tau)$ and radius $\bar{R}(\tau)$ of the rotor both increase with time. These results give a reasonably good picture of the qualitative features of the coupled system.

In Figure 3 is shown the effect of the non-linear term β_1 associated with the textile. It is observed that the amplitude is larger and the period of oscillation is shorter when the non-linearity is included. The curves were obtained by making the rotary angular velocity of the shaft angle to $-\frac{4}{3}\pi$.

In Figure 4 is shown the influence of shaft stiffness on the transient amplitudes. It is seen that increasing the shaft stiffness causes a decrease in both the transverse vibration of the textile (Figure 4(a)) and the whirling amplitude of the rotor (Figure 4(b)). The decrease in the whirling amplitude is much greater than that for the textile vibration.

In Figure 5 is shown the difference between the results for modes 1 and 2. It is observed that as the textile vibration mode number is increased, the coupling effect increases and the amplitudes are larger.

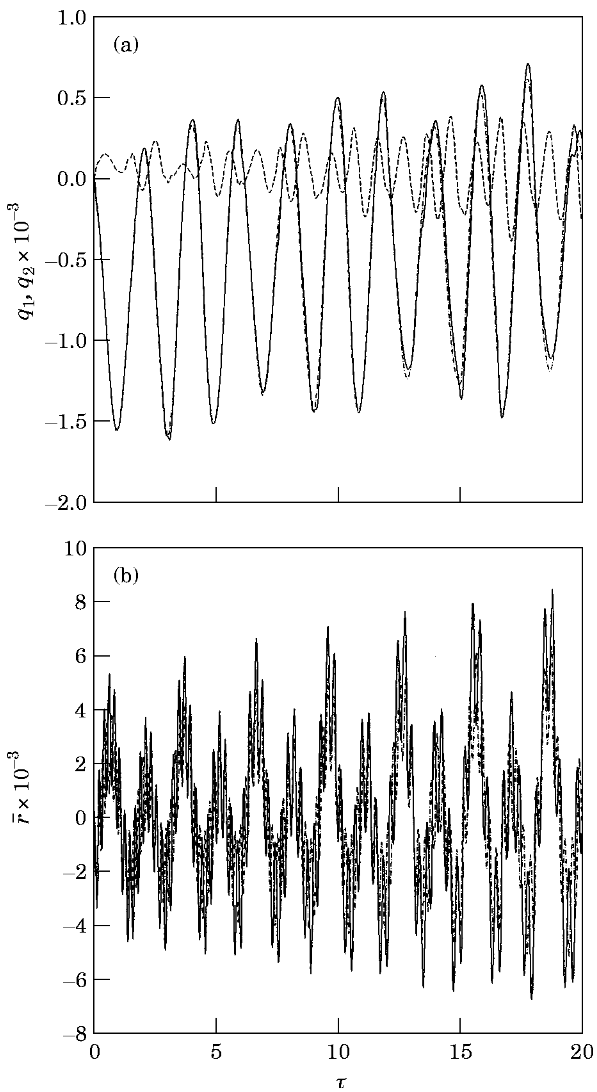

Figure 5. The influence of the shaft stiffness on the transient amplitudes for the case $\Omega_r^{(0)}/\Omega_1^{(0)} = 10$, $\Omega = -\frac{4}{3}\pi$. (a) The non-dimensional amplitude of the textile: ·—·—, q_1 for the mode = 1; ——, q_1; – – –, q_2 for mode = 2. (b) The non-dimensional amplitude of the rotor: ·—·—, mode = 1; ——, mode = 2.

5. CONCLUSIONS

A model of a coupled textile–rotor system that includes transverse textile vibration and rotor whirling has been formulated. In this paper, we assume that the instantaneous angular location of the imbalance with respect to the direction of shaft bending is constant, that the Jeffcott rotor exhibits synchronous whirling, and that the shaft angular speed remains constant. Numerical results have been presented for both linear and non-linear coupled systems. Our findings confirm that the amplitude is larger and the period of oscillation is shorter when the non-linearity is considered. The time-dependent mass and radius of the rotor, and the non-constant travelling velocity and time-dependent length of the textile, are also included. It is found that as the textile is wound on, the textile length decreases and its axial travelling velocity increases, and the mass and radius of the rotor both increase. These results give a reasonably good interpretation of the qualitative features of the coupled system. For future work, the non-synchronous vibration of the rotor, parametric excitation, the steady state response and dynamic stability analysis would be interesting problems to investigate.

ACKNOWLEDGMENT

The authors are greatly indebted to the National Science Council of the Republic of China for the support of this research through contract No. NSC 84-2212-E-033-012.

REFERENCES

1. G. V. Anand 1969 *Journal of the Acoustical Society of America* **45**, 1089–1096. Large-amplitude damped free vibration string of a stretched string.
2. J. A. Elliott 1982 *American Journal of Physics* **50**, 1148–1150. Nonlinearity in vibrating strings.
3. K. Yasuada and T. Torii 1986 *Bulletin of the Japan Society of Mechanical Engineers* **29**, 1250–1260. Nonlinear forced oscillations of a string.
4. K. R. Korde 1985 *Transactions of the American Society of Mechanical Engineers*, *Journal of Applied Mechanics* **52**, 493–494. Nonlinear oscillation of moving string.
5. J. A. Wickert and C. D. Mote Jr 1990 *Transactions of the American Society of Mechanical Engineers*, *Journal of Applied Mechanics* **57**, 738–744. Classical vibration analysis of axially moving continua.
6. K. W. Wang 1991 *Transactions of the American Society of Mechanical Engineers*, *Journal of Vibration and Acoustics* **113**, 62–68. Dynamic stability analysis of high speed axially moving bands with end curvatures.
7. K. W. Wang 1992 *Transactions of the American Society of Mechanical Engineers*, *Journal of Vibration and Acoustics* **113**, 119–126. On the stability of chain drive systems under periodic sprocket oscillations.
8. T. Kotera and R. Kawai 1988 *Nippon Kikai Gakksi Ronbunshu C Hen* **54**, 9–15. Vibrations of strings with time-varying length (2nd report, free vibrations induced by initial displacements).
9. R. F. Fung and W. H. Cheng 1993 *Journal of the Chinese Society of Mechanical Engineers* **12**, 229–239. Free vibration of a string/slider nonlinear coupling system.
10. L. Cventianin 1984 *Journal of Sound and Vibration* **97**, 181–187. Vibrations of a textile machine rotor.
11. L. Cventianin 1986 *Mechanism and Machine Theory* **21**, 29–32. The vibrations of a textile machine rotor with nonlinear characteristics.
12. L. Cventianin 1991 *Mechanism and Mechanic Theory* **26**, 253–260. The oscillations of a textile machine rotor on which the textile is wound up.
13. L. Cventianin 1993 *Journal of Sound and Vibration* **167**, 382–384. The influence of the reactive force on the motion of the rotor on which the band is winding up.
14. A. P. Bessonov 1974 *Osnovji Sinamiki Mehanizmoves Peremennojzven Zvenjev*. Moscow: Nauka.
15. L. Cventianin 1994 *Japan Society of Mechanical Engineers*, *International Journal* **37**, 41–48. Dynamic behavior of a rotor with time dependent parameters.

16. J. M. VANCE and J. LEE 1974 *Transactions of the American Society of Mechanical Engineers, Journal of Engineering for Industry*, August, 660–668. Stability of high speed rotors with internal friction.
17. P. K. C. WANG and J. WEI 1987 *Journal of Sound and Vibration* **116**, 149–160. Vibrations in a moving flexible robot arm.
18. K. YUH and T. YOUNG 1991 *Journal of Dynamic System, Measurement, and Control* **113**, 34–40. Dynamic modeling of an axially moving beam in rotation. simulation and experiment.

APPENDIX A

A.1. VIBRATION OF TEXTILE

From Figure 1, the position vector of any point on the textile after deformation is

$$\begin{aligned}\mathbf{R}_x(t) &= \mathbf{x} + \mathbf{w} \\ &= (x\cos\psi - w\sin\psi)\mathbf{i} + (x\sin\psi + w\cos\psi)\mathbf{j},\end{aligned} \tag{A1}$$

where **i** and **j** are unit vectors that point in the directions of increasing X and Y respectively $\mathbf{w} = \mathbf{w}(x, t)$ is the deflection of the textile at location **x**, and ψ is the rotary angle of the textile.

The Lagrangian function for the textile is the kinetic energy minus the potential energy. Thus, we have

$$\begin{aligned}L_s &= \tfrac{1}{2}\int_0^{l(t)} \rho\mathbf{V}\cdot\mathbf{V}\,\mathrm{d}x - \int_0^{l(t)} (T\varepsilon_E + \tfrac{1}{2}EA\varepsilon_E^2)\,\mathrm{d}x \\ &= \tfrac{1}{2}\left\{\rho\left[\dot{\psi}^2x^2 + 2\dot{\psi}^2x\frac{\mathrm{d}w}{\mathrm{d}t} + \left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)^2 + \dot{x}^2 - 2\dot{\psi}\dot{x}w + \dot{\psi}^2w^2\right] - Tw_x^2 - \tfrac{1}{4}EAw_x^4\right\},\end{aligned} \tag{A2}$$

where $\varepsilon_E = \frac{1}{2}w_x^2$ is the engineering strain, EA denotes the rigidity of the textile, $T\varepsilon_E$ and $\frac{1}{2}EA\varepsilon_E^2$ are, respectively, the terms due to initial tension and deflection. The latter is measured from the initially tensioned configuration.

A.2. WHIRLING ROTOR

The Lagrangian function for the rotor is

$$\begin{aligned}L_r = {}&\tfrac{1}{2}m(t)\{[\dot{r} - e\dot{\phi}\sin\beta^2]^2 + [r\dot{\phi} + e\dot{\phi}\cos\beta]^2\} + \tfrac{1}{2}I(t)\dot{\phi}^2 - \tfrac{1}{2}k_xr^2\cos^2\beta - \tfrac{1}{2}k_yr^2\sin^2\beta \\ &-m(t)g[r\sin(\phi+\beta) + e\sin(\phi+\beta)].\end{aligned} \tag{A3}$$

In order to derive the virtual work done by the initial tension T on the rotor, the virtual displacement at the connection point will be obtained first. The position vector of the connection point can be written as

$$\mathbf{R}_A(t) = (l_0 + r\cos\phi - R(t)\cos\theta)\mathbf{i} + (r\sin\phi + R(t)\sin\theta)\mathbf{j}. \tag{A4}$$

The virtual displacement of the connection point is

$$\delta\mathbf{R}_A(t) = (\delta r\cos\phi + R(t)\sin\theta\delta\theta)\mathbf{i} + (\delta r\sin\phi + R(t)\cos\theta\delta\theta)\mathbf{j}. \tag{A5}$$

Since θ is not the generalized co-ordinate chosen to describe the dynamic whirling, $\delta\theta$ should be replaced with δr. From Figure 1 it is seen that $\theta = \pi/2 - \psi$; hence we have $\delta\theta = -\delta\psi$. Consider the following geometrical relation:

$$\begin{aligned}\sin\psi &= \sin(\psi_1 + \psi_2)\\ &= \frac{R(t)}{a}\cos\psi_2 + \frac{r\sin\phi}{a}\cos\psi_1\\ &= \frac{1}{a^2}[R(t)(l_0 + r\cos\phi) + l(t)r\sin\phi],\end{aligned} \tag{A6}$$

where

$$a = \sqrt{l_0^2 + r^2 + 2l_0 r\cos\phi} \tag{A7}$$

is the auxiliary line. Taking the virtual angular displacement from equation (A6), we have

$$\delta\psi = C_r\delta r, \tag{A8}$$

where

$$\begin{aligned}C_r = {}&\frac{\left(R(t)\cos\phi + \dfrac{1}{l(t)}(r + l_0\cos\phi)r\sin\phi + l(t)\sin\phi\right)}{[l(t)(l_0 + r\cos\phi) - R(t)r\sin\phi]}\\ &- \frac{2(r + l_0\cos\phi)[R(t)(r + l_0\cos\phi) + l(t)r\sin\phi]}{a^2[l(t)(l_0 + r\cos\phi) - R(t)r\sin\phi]}.\end{aligned} \tag{A9}$$

Equation (A8) states the relationship between $\delta\psi$ and δr.

The initial tension vector is

$$\mathbf{T} = -T(\cos\psi\,\mathbf{i} + \sin\psi\,\mathbf{j}). \tag{A10}$$

The virtual work done by the initial tension can be expressed as

$$\begin{aligned}\delta W &= \mathbf{T}\cdot\delta\mathbf{l}(t)\\ &= T[\cos\psi\cos\phi - \sin\psi\sin\phi + C_r R(t)]\delta r.\end{aligned} \tag{A11}$$

A3. FORMULATION FOR THE COUPLED TEXTILE–ROTOR SYSTEM

To obtain the equations for the coupled system, the calculus of variations and Hamilton's principle are applied. However, the application of the principle is not straightforward, since there is a moving boundary involved at $x = l(t)$, where the position is not specified.

We consider the entire system including textile length $0 \leqslant x \leqslant l(t)$ and rotor Hamilton's principle can be written as

$$0 = \int_{t_1}^{t_2}\left[\delta\int_0^{l(t)} L_s(x, t; w, w_x, w_t)\,\mathrm{d}x + \delta L_r(t; r, \dot{r}) + \delta W\right]\mathrm{d}t \tag{A12}$$

where t_1 and t_2 are two arbitrary end times. In the process of taking the variation expressed by equation (A12), we apply the partial integration technique, using Leibnitz's theorem, and collect the like terms to obtain

$$0 = \int_{t_1}^{t_2} \left\{ L_s[l(t), t; w(l(t), t)]\delta l(t) + \int_0^{l(t)} \left(\frac{\partial L_s}{\partial w} - \frac{\partial}{\partial x}\frac{\partial L_s}{\partial w_x} - \frac{\partial}{\partial t}\frac{\partial L_s}{\partial w_t} \right) \delta w \, \mathrm{d}x \right.$$

$$+ \left[\left(\frac{\partial L_s}{\partial w_x} - \frac{\mathrm{d}x}{\mathrm{d}t}\frac{\partial L_s}{\partial w_t} \right) \delta w \right]_{x=0}^{x=l(t)} + \left(\frac{\partial L_r}{\partial r} - \frac{\partial}{\partial t}\frac{\partial L_r}{\partial \dot{r}} + T[\cos\psi\cos\phi - \sin\psi\sin\phi \right.$$

$$\left. \left. + C_r R(t)] \right) \delta r \right\} \mathrm{d}t + \left[\int_0^{l(t)} \frac{\partial L_s}{\partial w_t}\delta w \, \mathrm{d}x + \frac{\partial L_s}{\partial \dot{r}}\delta r \right]_{t_1}^{t_2}. \qquad \text{(A13)}$$

The varied path coincides with the true path at the two end points t_1 and t_2. It follows that $\delta w(t_1) = \delta w(t_2) = 0$ and $\delta r(t_1) = \delta r(t_2) = 0$. In the variational process, $\delta l(t)$ exists in equation (A13) because the position $x = l(t)$ is not specified. Taking the variation with respect to equation (1), we have

$$\delta l(t) = \frac{1}{l(t)}(r + l_0\cos\phi)\delta r. \qquad \text{(A14)}$$

Substituting equation (A14) into the first term of equation (A13), and collecting with the

$$\left(\frac{\partial L_r}{\partial r} - \frac{\partial}{\partial t}\frac{\partial L_r}{\partial \dot{r}} \right)\delta r$$

term, we can obtain Lagrange's equations for the textile and rotor, respectively, as

$$\frac{\partial L_s}{\partial w} - \frac{\partial}{\partial x}\frac{\partial L_s}{\partial w_x} - \frac{\partial}{\partial t}\frac{\partial L_s}{\partial w_t} = 0, \qquad 0 < x < l(t), \qquad \text{(A15)}$$

$$\frac{\partial L_r}{\partial r} - \frac{\partial}{\partial t}\frac{\partial L_r}{\partial \dot{r}} + T[\cos\psi\cos\phi - \sin\psi\sin\phi] + \frac{1}{l(t)}(r + l_0\cos\phi)L_s[l(t), t; w(l(t), t)] = 0, \qquad \text{(A16)}$$

where the boundary conditions are

$$w(0, t) = 0, \qquad w(l(t), t) = 0. \qquad \text{(A17, A18)}$$

Substituting the Lagrangian functions (A2) and (A3) for the textile and rotor, respectively, into equations (A15) and (A16), and considering the rotor with angular velocity $\dot{\phi} = \omega =$ constant, we obtain the governing equations for the system as

$$w_{tt} + 2\dot{x}w_{xt} + \ddot{x}w_x - \dot{\psi}^2 w - \left(\frac{T}{\rho} - \dot{x}^2 \right) w_{xx} - \tfrac{3}{2}\frac{EA}{\rho}w_x^2 w_{xx} = -3\dot{x}\dot{\psi} - x\ddot{\psi}, \qquad 0 < x < l(t), \qquad \text{(A19)}$$

$$\ddot{r} + \left[\frac{k_x}{m(t)}\cos^2\beta + \frac{k_y}{m(t)}\sin^2\beta - \omega^2\right] r = e\omega^2\cos\beta + \dot{e}\omega\sin\beta - g\sin\phi$$

$$- \frac{T}{m(t)}[\cos\psi\cos\phi - \sin\psi\sin\phi]$$

$$- \frac{(r + l_0\cos\phi)}{2m(t)l(t)}[\rho\dot{x}^2 + \rho\dot{x}^2 w_x^2(l(t), t)$$

$$- Tw_x^2(l(t), t) - \tfrac{1}{4}EAw_x^4(l(t), t). \qquad \text{(A20)}$$

APPENDIX B

The time-varying coefficients of equation (14a) are as follows:

$$A_{mn}(\bar{l}, \dot{\bar{l}}) = \int_0^{\bar{l}} \dot{\varphi}_n \varphi_m \,\mathrm{d}\xi, \qquad B_{mn}(\bar{l}) = \int_0^{\bar{l}} \varphi_n' \varphi_m \,\mathrm{d}\xi,$$

$$C_{mn}(\bar{l}) = \xi_{\tau\tau}\int_0^{\bar{l}} \varphi_n' \varphi_m \,\mathrm{d}\xi, \qquad D_{mn}(\bar{l}, \dot{\bar{l}}, \ddot{\bar{l}}) = \int_0^{\bar{l}} \ddot{\varphi}_n \varphi_m \,\mathrm{d}\xi,$$

$$E_{mn}(\bar{l}, \dot{\bar{l}}) = \int_0^{\bar{l}} \dot{\varphi}_n' \varphi_m \,\mathrm{d}\xi, \qquad F_{mn}(\bar{l}) = \int_0^{\bar{l}} \varphi_n'' \varphi_m \,\mathrm{d}\xi,$$

$$N_{mijk}(\bar{l}, \dot{\bar{l}}, \ddot{\bar{l}}) = \int_0^{\bar{l}} \varphi_i' \varphi_j' \varphi_k'' \varphi_m \,\mathrm{d}\xi.$$

The coefficients of equation (14b) are as follows:

$$a^r(\bar{l}) = \Omega_x^2\cos^2\beta + \Omega_y^2\sin^2\beta - \Omega^2 - \frac{M}{2}\bar{l}\dot{\psi}^2 + \frac{M}{2\bar{l}}\dot{\xi}^2 - c_{ni1}^r(\bar{l}) - d_{ni1}^r(\bar{l}) - e_{nijk1}^r(\bar{l}),$$

$$b_{ni}^r(\bar{l}) = -M\dot{\xi}\cos\phi\,\frac{n\pi}{\sqrt{\bar{l}^3}}(-1)^n\dot{\psi}, \qquad c_{ni}^r(\bar{l}) = -\frac{M\dot{\xi}^2\cos\phi}{2\bar{l}}\frac{2ni\pi^2}{\bar{l}^3}(-1)^{(n+i)},$$

$$d_{ni}^r = \frac{M\cos\phi}{2\bar{l}}\frac{2ni\pi^2}{\bar{l}^3}(-1)^{(n+i)}, \qquad e_{nijk}^r(\bar{l}) = \frac{M\cos\phi\beta_1^2}{8\bar{l}}\frac{4nijk\pi^4}{\bar{l}^6}(-1)^{(n+i+j+k)},$$

$$f^r(\bar{l}) = \bar{e}\Omega^2\cos\beta - \bar{g}\sin\phi - \frac{M\cos\phi}{2}\bar{l}\dot{\psi}^2 - \frac{M\cos\phi}{2\bar{l}}\dot{\xi}^2$$

$$- M(\cos\psi\cos\phi - \sin\psi\sin\phi - \bar{C}_r\bar{R}),$$

where

$$c_{ni1}^r(\bar{l}) = -\frac{M\dot{\xi}^2}{2\bar{l}}\frac{2ni\pi^2}{\bar{l}^3}(-1)^{(n+i)}, \qquad d_{ni1}^r(\bar{l}) = \frac{M}{2\bar{l}}\frac{2ni\pi^2}{\bar{l}^3}(-1)^{(n+i)},$$

$$e_{nijk1}^r(\bar{l}) = \frac{M\beta_1^2}{8\bar{l}}\frac{4nijk\pi^4}{\bar{l}^6}(-1)^{(n+i+j+k)}.$$